Exercise 6

With the aid of relations (10) and (11) in Sec. 3, derive the identity

$$\left(\frac{z_1}{z_3}\right)\left(\frac{z_2}{z_4}\right) = \frac{z_1 z_2}{z_3 z_4} \qquad (z_3 \neq 0,\ z_4 \neq 0).$$

Solution

Relations (10) and (11) in Sec. 3 state that for two complex numbers, z_1 and z_2,

$$\frac{z_1}{z_2} = z_1\left(\frac{1}{z_2}\right) \qquad (z_2 \neq 0) \tag{10}$$

$$\left(\frac{1}{z_1}\right)\left(\frac{1}{z_2}\right) = z_1^{-1} z_2^{-1} = (z_1 z_2)^{-1} = \frac{1}{z_1 z_2} \qquad (z_1 \neq 0,\ z_2 \neq 0). \tag{11}$$

Apply relation (10) twice.

$$\begin{aligned}\left(\frac{z_1}{z_3}\right)\left(\frac{z_2}{z_4}\right) &= z_1\left(\frac{1}{z_3}\right) z_2\left(\frac{1}{z_4}\right) \\ &= z_1 z_3^{-1} z_2 z_4^{-1}\end{aligned}$$

Use the commutative property for multiplication of complex numbers.

$$= z_1 z_2 z_3^{-1} z_4^{-1}$$

Use relation (11).

$$\begin{aligned}&= z_1 z_2 (z_3 z_4)^{-1} \\ &= z_1 z_2\left(\frac{1}{z_3 z_4}\right)\end{aligned}$$

Use relation (10) once again.

$$= \frac{z_1 z_2}{z_3 z_4}$$

z_3 and z_4 are assumed to be nonzero, of course.